Radiation Physics
General Math Review
Objective

› The idea behind this presentation is to provide a basic overview of the kind of math you may expect to encounter in PHY-106.
  - So if you have math anxiety, that’s okay. Hopefully this will help you understand what you need to know in order to do well.

› I also want to demonstrate real examples of material in the class that will use these math concepts.
  - That way, you can see the practical application.
  - It’s not important at this point that you understand the physics concepts. I just want you to see how the math is used.

› This presentation works best if you go full-screen. I spent a lot of time on the animations. Please enjoy them.
General Math Topics

› Order of Operations
› Ratio and Proportion
› Fractions and Percent
› Bases and Exponents
› Graphs and Charts
› Scientific Notation
› Units
› Dimensional Analysis
Order of Operations

- PEMDAS – Parentheses, Exponents, Multiplication and Division, Addition and Subtraction

\[ 3(5 + 8) - \frac{2^2}{4} + 3 \]

\[ 3(13) - \frac{4}{4} + 3 \]

\[ 39 - 1 + 3 = 41 \]
Order of Operations

› Distributive Property:

\[ a(b + c) = ab + ac \]

\[ 3(x + 2) = 3x + 6 \]
Equations

\[ x + 4 = 7 - 3 + 4 \]
\[ x + 4 = 8 \]
\[ -4 - 4 \]
\[ x = 4 \]
Equations

\[
\begin{align*}
6 + 3 - 1 &= 4 + 1 - a \\
8 &= 5 - a \\
-5 - 5 &= 3 = \neq a \\
-1 &= 1 \\
\end{align*}
\]

\[a = -3\]
Equations

\[ \beta \cdot \frac{x}{\beta} = \frac{7}{9} \cdot 3 \]

\[ x = \frac{21}{9} = \frac{7}{3} \]
Equations

\[ \frac{4}{y} = \frac{6 + 3}{10} \]

\[ \frac{4}{y} \times \frac{9}{10} = \frac{40}{9} \]

Cross-multiply

\[ \frac{9y}{9} = \frac{40}{9} \]

\[ y = \frac{40}{9} \]
Equations

\[
\frac{x + 4}{8} = 7 + 2 - 3
\]

\[
8 \cdot \frac{x + 4}{8} = 6 \cdot 8
\]

\[
x + 4 = 48
\]

\[
\underline{-4} - 4
\]

\[
x = 44
\]
Ratio and Proportion

› A **ratio** is a fixed relationship between two quantities, simply indicating how many times larger or smaller one quantity is, relative to another.

› What does that mean? If a cake recipe calls for 2 cups of sugar and 1 cup of milk, there is a 2:1 sugar-to-milk ratio.

› Ratios can also be represented as a fraction: \( \frac{a}{b} \)

› Finally, you may also see ratios represented as a single number, rather than as two numbers separated by a colon or a fraction: 

\[
1:2 = \left( \frac{1}{2} \right) = 0.5
\]

› So pay attention to whether your ratio number is greater or less than 1!
Example Using a Transformer

- A “turns ratio” shows the ratio between the number of copper wire turns on the secondary side of a transformer vs. the primary side.
  
  \[ TR = \frac{N_s}{N_p} \]

- The “turns ratio” is 6:16, which is the same as 6/16, which gives a turns ratio of 0.375.

- Because the turns ratio is less than 1, this is a step-down transformer.
Proportionality

Related to the concept of ratio, a **proportion** simply means that two given ratios are equal. It’s a way of relating one variable to another.

There are two ways to demonstrate proportionality, and both are used in this class (I’ll give examples of each in the coming slides):

- As an equivalence of ratios, which means your problem will look like this:
  \[
  \frac{a_1}{b_1} = \frac{a_2}{b_2}
  \]

- As a relationship with a **proportionality constant**, which means it’ll look like this:
  \[
  y = kx
  \]

There are also two types of proportionality:

- Direct
- Indirect
Direct Proportion

› Remember that proportion relates 2 variables.

› Simply stated, **direct proportion** means that as one variable increases, the other also increases, and vice versa.
  
  • If the ratio between 2 factors is 1:2 (1/2, or .5), and the first factor is doubled (1x2), then the second factor will be quadrupled (2x2); but both of them increase, and *that’s* what makes it a direct proportion.

› You can recognize a direct proportion by the way the formula is constructed. Direct proportions look like this:
  
  • \( \frac{a_1}{b_1} = \frac{a_2}{b_2} \)
  
  • \( y = kx \)
Examples of Direct Proportion

› Voltage in a Transformer (Transformer Law):

\[
\frac{V_s}{V_p} = \frac{N_s}{N_p}
\]

› Particle Theory of Light:

\[E = h\nu\]

- Constant (in this case, Planck’s constant)
- y-variable
- x-variable
Inverse Proportion

- One variable varies in the opposite direction from the other variable as it changes (another way of saying this is that the first factor varies directly with the inverse $\frac{1}{b}$ of the second).

- Or, simply put, if one variable increases, the other decreases, and vice versa. You can recognize it in one of two ways:
  - As an equivalence of ratios:
    \[
    \frac{a_1}{\frac{1}{b_1}} = \frac{a_2}{\frac{1}{b_2}} \rightarrow a_1 b_1 = a_2 b_2 \rightarrow \frac{a_1}{a_2} = \frac{b_2}{b_1}
    \]
  - With a proportionality constant:
    \[
    y = k \left(\frac{1}{x}\right) \rightarrow y = \frac{k}{x}
    \]

Notice the 2's and the 1's are flipped.
Examples of Indirect Proportion

› Inverse Square Law:

\[
\frac{I_1}{I_2} = \frac{(D_2)^2}{(D_1)^2}
\]

› Wave Theory of Light:

\[
c = \lambda \nu \quad \Rightarrow \quad \lambda = \frac{c}{\nu}
\]

Hint: “c” is your proportionality constant
Exponent Laws

› When multiplying two numbers with exponents and the *same* base, keep the base and add the exponents:
\[ B^m \cdot B^n = B^{m+n} \]
\[ 10^5 \cdot 10^2 = 10^{5+2} = 10^7 \]

› When dividing with the same base, subtract the exponents:
\[ \frac{B^m}{B^n} = B^{m-n} \]
\[ \frac{10^5}{10^2} = 10^{5-2} = 10^3 \]

› If you were to raise an exponent to an exponent, multiply the exponents together:
\[ (B^m)^n = B^{m\cdot n} \]
\[ (10^5)^2 = 10^{5\cdot 2} = 10^{10} \]
Graphs and Charts

› Graphs and charts are visual diagrams that visually demonstrate the relationship between two variables, one of which depends on the other.
  
  – The variable that we believe is *causing* the change in the other factor is called the **independent variable**, and is placed on the *x*-axis.
  
  – The variable that *is changed* by the independent variable is the **dependent variable**, which is placed on the *y*-axis.

› When a relationship is found, the *y*-axis (dependent) variable is said to **be a function of** the *x*-axis (independent) variable.
This is an example of a problem you can expect to see:
How long would it take for the tube to cool completely after generating 120,000 heat units? (8 mins – 2 mins = 6 mins)
How long would it take to cool from 150,000 units to 50,000? (5 mins – 1 min = 4 mins)
Scientific Notation

› This is used for representing very large and very small numbers.

› Rules for Proper Scientific Notation:
  – Only one integer can go in front of the decimal point.
  – Have as many integers following the decimal point as are required by significant figures (most of the time, people usually use 3 sig figs, so there would be 2 digits after the decimal).
  – This number is then multiplied by 10 raised to an exponent.
  – Example:
    \[ 3.21 \times 10^{-15} \]
Features of Scientific Notation

- When 10 is raised to a positive exponent, the number becomes larger. When 10 is raised to a negative exponent, the number becomes smaller.
  - A negative exponent does not indicate a negative number.

- When the decimal point is not in the correct location, you will have to move it and adjust the exponent correspondingly.
  - If the decimal point needs to be moved to the right, the exponent will go to the left on the number line:

$$0.0052 \times 10^8 \rightarrow 5.2 \times 10^{8-3} \rightarrow 5.2 \times 10^5$$
Multiplying in Scientific Notation

Don’t try putting the number directly into your calculator! It may not work (especially if dividing)! Instead:

\[(1.60 \times 10^{15})(1.875 \times 10^{-7})\]

Separate the problem into two parts:

\[(1.60 \times 1.875) = 3.00\]

(10^{15} \times 10^{-7})

= 10^{15+(-7)} = 10^8

Put it back together:

\[3.00 \times 10^8\]
Dividing in Scientific Notation

› Same Strategy:

\[
\frac{(3.00 \times 10^8)}{(4.6875 \times 10^{16})}
\]

\[
\left(\frac{3.00}{4.6875}\right) = 0.640
\]

\[
= 10^{8-16} = 10^{-8}
\]

Is that proper scientific notation though? **Of course not!**

› The decimal point needs to go to the *right* by one space, so the exponent goes to the *left* on the number line one space.

\[
6.40 \times 10^{-9}
\]
Fundamental Units

› Two major systems of measurement: English and metric
  – English is garbage
  – Metric is awesome

› Metric system has lots of names, but they:
  – Systéme Internationale (SI)
  – MKS (meter-kilogram-second)
  – CGS (centimeter-gram-second)

› Derived units are compounds of fundamental units:
  – Examples: Area (m²), Volume (m³), Density (m/V), Velocity (m/s)

› Get comfortable with manipulating units using dimensional analysis.
Fundamental Units

Here’s how dimensional analysis works. Start by writing down what you’re given.

- Then set up a fraction where the numerator and denominator are equal to each other, but with different units.
- Make sure the denominator has the same unit as what you’re starting with (so the units cancel out)
- Multiply by the numerator, then divide by the denominator.

Here’s what a problem might look like. Convert 1 kilometer to miles (without using Google, you dirty cheaters):

\[ 1 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = 0.621 \text{ mi} \]

- You won’t get a problem this complicated, but this shows you the general form.
Any Questions?

› Okay, “Calc”-you-later!!!